that as an example, hydrogen gas, compared with atmospheric air, appears to possess this property in a proportion more than double that which would be given by the respective densities of air and hydrogen gas.

The paper concludes with an investigation of the effect which the near reduction to a vacuum will have on the variations of gravity at different parts of the earth's surface, which have been obtained with invariable pendulums; and particularly of the experiments of the author himself, which embrace a greater range and variety of temperature than those of any other experimentalist. It is shown that in consequence of the peculiar mode in which those experiments were reduced to a mean term of comparison, their re-calculation with the more correct elements now known, would have no other effect than that of adding an equal amount to the vibrations of the experimental pendulum at every station, leaving the acceleration at different stations unaltered.

Consideration of the Objections raised against the geometrical Representation of the Square Roots of Negative Quantities. By the Rev. John Warren, *M.A. of Jesus College, Cambridge. Communicated by* Thomas Young, *M.D. For. Sec. R.S.* Read February 19, 1829. [*Phil. Trans.* 1829, *p.* 241.]

It has always appeared a paradox in mathematics, that by employing what are called imaginary or impossible quantities, and subjecting them to the same algebraic operations as those which are performed on quantities that are real and possible, the results obtained should always prove perfectly correct. The author inferring from this fact, that the operations of algebra are of a more comprehensive nature than its definitions and fundamental principles, was led to inquire what extension might be given to these definitions and principles, so as to render them strictly applicable to quantities of every description, whether real or imaginary. This deficiency, he conceives, may be supplied by having recourse to certain geometrical considerations. By taking into account the directions as well as the lengths of lines drawn in a given plane, from a given point, the addition of such lines may admit of being performed in the same manner as the composition of motions in dynamics; and four such lines may be regarded as proportional, both in length and direction, when they are proportionals in length, and, when also the fourth is inclined to the third at the same angle that the second is to the first. From this principle he deduces, that if a line drawn in any given direction be assumed as a positive quantity, and consequently its opposite a negative quantity, a line drawn at right angles to the positive or negative direction will be represented by the square root of a negative quantity; and a line drawn in an oblique direction will be represented by the sum of two quantities, the one either positive or negative, and the other the square root of a negative quantity. On this subject, the author published a treatise in April 1828; since

The Royal Society is collaborating with JSTOR to digitize, preserve, and extend access to *Abstracts of the Papers Printed in the Philosophical Transactions of the Royal Society of London.* www.jstor.org

which period several objections have been made to this hypothesis. The purpose of the present paper is to answer these objections.

The first of these is, that impossible roots should be considered merely as the indications of some impossible condition, which the proposition that has given rise to them involves; and that they have in fact no real or absolute existence. To this it is replied by the author, that although such a statement may be true in some cases, it is by no means necessarily so in all; and that these quantities resemble in this respect fractional and negative roots, which, whenever they are excluded by the nature of the question, are indeed signs of impossibility, but yet in other cases are admitted to be real and significant quantities. We have therefore no stronger reasons, *à priori*, for denying the real existence of what are called impossible roots, because they are in some cases the signs of impossibility, than we should have for refusing that character to fractional or negative roots on similar grounds.

It has been objected, in the second place, that there is no necessary connexion between algebra and geometry, but only one of analogy; and that it is consequently improper to introduce geometric considerations into questions purely of an algebraic nature. In answer to this, the author contends that a necessary connexion may be shown to exist between impossible roots, and the series expressive of the ratio between the circumference of a circle and its diameter. This he endeavours to prove by examining such values of the expansion of 1^x as are functions of x; whereby he is led to a series, the terms of which involve both the square root of unity, and also the above-mentioned geometric ratio. In other cases he arrives, by methods which are purely algebraic, to expressions containing sines and cosines, together with impossible roots. Hence the author infers that a necessary connexion exists between algebra and geometry; and that his own hypothesis as to the geometric representation of the square roots of negative quantities, is true in the same sense as the hypothesis adopted by algebraists respecting the geometric representation of negative quantities is true.

To a third objection, derived from the alleged inutility of such a geometric representation of the square roots of negative quantities, the author replies, that from their frequent employment by mathematicians, it is reasonable to expect that they will be of much greater use when the true theory of their nature shall be established than when it was unknown.

If the hypothesis of the author is admitted, all questions in dynamics where the motions of bodies are limited to one plane, will be brought within the province of pure algebra.

The author concludes by noticing a work by M. Mourey, entitled "*La vraie Théorie des Quantités Négatives, et des Quantités prétendues Imaginaires*," in which the same general views of the subject are presented as are entertained by the author.